Errata and Addenda in "The Structure of Compact Groups" 3rd Edition of 2013

Version of May 20 , 2019

page xx	Line 7 from below: Replace "Degree" by "Rank".
page 23	Lines 6,7 of Exercise E1.12 read: (A subgroup of a topological group is called *characteristic*, if it is invariant under all (continuous and continuously invertible!) automorphisms. It is called *fully characteristic*, if it is invariant under all (continuous!) endomorphisms.)
page 29	Line 4 below Table 1.2 read: "For $\mathbb{Z}_p$ and $\mathbb{Z}(p^\infty)$, see Example 1.38(i)."
page 29	Line 5 below Table 1.2 read: "For $\mathbb{T}_p$ and $\frac{1}{p^\infty}\mathbb{Z}$, see Example 1.38(ii)."
page 49	Line 11 from below: Replace "geomestric" by "geometric".
page 55	Line 10: Replace "$C(G, K)$" by "$C(G, \mathbb{K})$".
page 70	Line after Exercise E3.13: Replace "[Hint. Read . . . .]" by the following:

[Hint: Let U_0 be an arbitrary closed neighborhood of 0 in V. It suffices to show that a finite union of translates of U_0 covers K. We find a closed convex neighborhood U of O such that $U + U \subseteq U_0$. Since P is precompact, there is a finite subset Q of P such that $P \subseteq Q + U$. The convex hull

$$X = \{\sum_{x\in Q} r_x{\cdot}x : 0 \le r_x, x \in Q, \text{ and } \sum_{x\in Q} r_x = 1\}$$

of Q is compact and $X + U$ is closed and convex, and $P \subseteq Q + U \subseteq X + U$, hence $X + U$ contains K. By the compactness of X there is a finite subset R of X such that $X \subseteq R + U$. Thus $K \subseteq X + U \subseteq R + U + U \subseteq R + U_0$.]

page 75	Line 3: Delete "X_{fix}".
page 84	Line 6: Replace "$\kappa^2 = \kappa$" by "$\kappa^2 = \mathrm{id}_{E_\mathbb{C}}$".
page 132	Line 6 above Theorem 5.27: Replace "$r\mathfrak{s}.x = \ldots$" by "$r{\cdot}x = \ldots$".
page 163	Line 2 from below read: "$j = 1, \ldots, n$, form a basis" [insert comma].
page 164	Line 4: Replace "X_1" by "X_1'" and "X_n" by "X_n'"

page 164	Line 1 below displayformula $(*)$: Replace "$]-\varepsilon_0,\varepsilon[$" by "$]\varepsilon_0,\varepsilon_0[^n$".
page 167	Line 8: Replace "U_g" by "U_h".
page 167	Lines 7 from below to last line: Replace these lines by the following text:

$f(\exp r_j{\cdot}X_j){\in}W_H$. Define a group homomorphism $\tau\colon\mathbb{R}{\to}H$ by $\tau(r){=}f(\exp r{\cdot}X_j)$. If τ is constant, set $r_j=\frac{1}{2}$. In that case $f(\exp r_j{\cdot}X_j)=1\in W_H$. Now assume that τ is nondegenerate. Then the subset $\tau(]0,1])$ of the compact space H is infinite and therefore has an accumulation point h. Let V be an identity neighborhood of H such that $VV^{-1}\subseteq W_H$. Find two real numbers s and t such that $0<s<t\leq 1$, that $\tau(s)\neq\tau(t)$, and that $\tau(s),\tau(t)\in Vh$. We set $r_j=t-s$. Then $0<r_j\leq 1$ and $f(\exp r_j{\cdot}X_j)=\tau(r_j)=\tau(t)\tau(s)^{-1}\in Vhh^{-1}V^{-1}=VV^{-1}\subseteq W_H$. Thus r_j satisfies our requirements. $\square$

page 180	Line 1 from below: Replace "F is a..." by "Z is a...".
page 199	Theorem 6.10: The text of the statement of Theorem 6.10 should be typeset in italics.
page 211	Line 15: replace "$N(G,T)$" by "$N(T,G)$".
page 220	Line 8 from below: replace "Z" by "H".
page 221	Line 19: replace "Z_0" by "H".
page 221	Line 20: replace "G'" by "N" and read "cofactor of N and..."
page 237	Line 4 from below: Replace "Theorem 6.47" by "Proposition 6.47"
page 255	Lines 8, 9: delete "$\mathrm{Out}(\mathfrak{g})\cong\mathrm{O}(3)/\mathrm{SO}(3)\cong\mathbb{Z}(2)$."
page 273	Line 8 from below: delete "$=\mathrm{O}(3)$"
page 273	Line 5 from below: replace "with O(3)" by "with SO(3)"
page 273	Line 4 from below: Replace "$\mathrm{diag}(\pm1,\pm1,\pm1)$" by "$\mathrm{diag}(\pm1,\pm1,\pm1)$ of determinant 1".
page 273	Line 3 from below: replace "$\mathrm{diag}(\pm1,1,1)$" by "$\mathrm{diag}(1,1,1)$, $\mathrm{diag}(-1,-1,1)$".

page 329	Line 4: replace “By (iv)” by “By (iii)”.
page 366	Line 1 of Corollary 7.68: Replace “charactergroup” by “character group”.
page 383	Line 9: replace “set X” by “set $X \neq \emptyset$”.
page 383	Line 11: replace “is a compact” by “is a nonsingleton compact”.
page 388	Line 16: replace “$\dim_Q$” by “$\dim_{\mathbb{Q}}$”.
page 401	above Section headline “Local Connectivity” insert: For more information on arc components see Part 6 of this Chapter under “Arc Components and Borel Sets”. (Numbers 8.86–8.99, pp. 445–449.)
page 446	Line 2 of the proof of Lemma 8.89: replace “S8.2” by “8.87”.
page 448	Lines 13, 14 read: Thus Theorem 8:30 and Proposition 8.97 motivate us to formulate the following statement:
page 448	Line 8 from below: Replace “$\mathrm{Ext}(A, Z)$” by “$\mathrm{Ext}(A, \mathbb{Z})$”.
page 449	Line 6 from below: Remove period between “set” and “and”.
page 449	Line 5 from below: Remove period between “l” and “d”.
page 450	Line 3: Replace “G” by “$\widehat{G}$”.
page 450	Line 6: Replace the entire line by: “Therefore $\mathfrak{L}(G)/\mathfrak{K}(G) \cong G_a$ where $\mathfrak{K}(G) = \ker \exp \cong \pi_1(G)$, and this also provides”
page 473	Line 1 from below (and p. 474 line 1) read: “… i.e. each of them is mapped into itself by all continuous endomorphisms.”
page 477	Lines 12, 15, 16 (counting headline): replace “$N(G,T)$” by “$N(T,G)$”.
page 501	Line 5 from below: Line should begin “(ii) If $\dim G < \infty$”
page 506	Line 13: replace “$\subseteq=$” by “$\subseteq$”.
page 520	Line 7: replace $\mathbb{Q}$ by $\widehat{\mathbb{Q}}$.

page 532	Line 3 of proposition 9.85 read: " the subgroup $\mathrm{Aut}(A)$ of $P(A)$ by the topology of $D(A)$." [Insert "by".]
page 533	Item (iii) of Theorem 9.86, read: $\mathrm{Aut}\left(S_{[\mathfrak{s}]}\right)^{X_\mathfrak{s}}$
page 533	Item (iv) of Theorem 9.86, line 1 read: $\left[\mathrm{Aut}\left(S_{\vert\mathfrak{s}\vert}\right)\right]_0 D_\mathfrak{s}$ [Delete (.]
page 533	Item (iv) of Theorem 9.86, line 2 read: $N\left(T_{[\mathfrak{s}]}, \mathrm{Aut}(S_{[\mathfrak{s}]})\right) = \{\alpha \in \mathrm{Aut}\left(S_{\vert\mathfrak{s}\vert}\right) \ldots\}$ [Insert one), delete one).]
page 536	Line 3 of the proof of Theorem 9.90 read: "small closed normal subgroups $N \in \mathcal{N}(G)$. It follows that G must be a Lie group."
page 536	Line 4 from below read: "and this proves $N = G$ which..." (delete isolated "s").
page 568	Line 9: Replace "(see 2.71)" by "(see 2.17)".
page 568	Line 11: Replace "sense of 6.77(ii)" by "sense of 10.29".
page 575	Line 4 of the proof of 10.41 read: "in particular for the case that $G/N \ldots$"
page 600	Line 12 from below read: $p(x)$ (not $\mathfrak{p}(x)$).
page 600	Line 6 from below read: $p \circ \widetilde{f} = f$ (not $p \circ F = f$).
page 600	Line 4 from below read: "Skljarenko" (not "Sklarjenk").
page 644	Line 1 of the proof of 11.58: Replace "N compact" by "N the compact"
page 652	Line 3 (headline): Replace "Degree" by "Rank".
page 654	Line 5: Replace "degree" by "rank".
page 654	Line 8: Replace "A1.20" by "A1.21".
page 656	Line 8: Replace "degree" by "rank".
page 659	Line 7 from below: Replace "degree" by "rank".
page 660	Line 13: Replace "degree" by "rank".

page 661 Line 5: Replace "degree" by "rank".

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page 667 Lines 5, 18 after the headline "Postscript": Replace "degree" by "rank".

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page 686 Line 17 from below: Replace "$\cdots \in A$" by "$\cdots \in \nabla$".

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page 686 Line 4 from below: Read

(iv) $\ker p \operatorname{tor}(\nabla)$. That is, $0 \to \operatorname{tor}(\nabla) \to \nabla \xrightarrow{p} \mathbb{Q} \to 0$ is exact.

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page 687 before *Proof* insert:

(x) $\operatorname{tor}(\nabla) \cong \bigoplus_{\mathrm{n}=2}^{\infty} \mathbb{Z}(\mathrm{n})$*, and there is an exact sequence*

$$0 \to \bigoplus_{n=2}^{\infty} \mathbb{Z}(n) \to \nabla \xrightarrow{p} \mathbb{Q} \to 0.$$

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page 687 Line 1 of *Proof*: Replace "$Z^{(\mathbb{N})}$" by "$\mathbb{Z}^{(\mathbb{N})}$".

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page 688 Line 6: Replace "$K = \ldots$" by "$S = \ldots$".

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page 688 Line following first displayline, read:

Since there is an isomorphism $\kappa\colon \bigoplus_{n=2}^{\infty} \mathbb{Z}(n) \to \nabla/\mathbb{Z}{\cdot}g_1$ by (v), the first assertion follows if we define

$$K \stackrel{\text{def}}{=} \kappa^{-1}(\nabla_1/\mathbb{Z}{\cdot}g_1).$$

In order to prove

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page 688 Lines 11 and 10 from below: Replace "$\mathbb{N}_0^{\mathbb{N}}$" by "$\mathbb{N}_0^{(\mathbb{N})}$".

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page 688 Lines 1 from below: Replace "$p^N\mathbb{Z}$" by "$p^n\mathbb{Z}$".

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page 689 Before the end-of-proof box insert:

(x) The second part follows from the first through (ii) and (iv). For proving the first part, we show that there is an injective endomorphism $\eta\colon S \to S$ with image K.

For this purpose we invoke (ix) to see that it is sufficient to show that for each prime p, there is an exact sequence

$$(E) \qquad 0 \to \bigoplus_{n=1}^{\infty} \mathbb{Z}(p^n) \xrightarrow{\eta_p} \bigoplus_{n=1}^{\infty} \mathbb{Z}(p^n) \to \mathbb{Z}(p^{\infty}) \to 0.$$

Indeed we define $\eta_p(\varepsilon_n) = p{\cdot}\varepsilon_{n+1} - \varepsilon_n$, $n = 1, 2, \ldots$. Since clearly $p{\cdot}\varepsilon_{n+1} - \varepsilon_n$ has order p^n, we have to show that η_p is injective. For a proof let $a = \sum_{n=1}^{\infty} z_n{\cdot}\varepsilon_n$

with a finite support sequence of elements $z_n \in \mathbb{Z}$ such that $\eta_p(a) = 0$. Then $0 = \sum_{n=1}^{\infty} z_n{\cdot}\varepsilon_n - \sum_{n=1}^{\infty} pz_n{\cdot}\varepsilon_{n+1} = \sum_{n=1}^{\infty} z_n{\cdot}\varepsilon_n - \sum_{n=2}^{\infty} pz_{n-1}{\cdot}\varepsilon_n = z_1.\varepsilon_1 + \sum_{n=2}^{\infty}(z_n - pz_{n-1}){\cdot}\varepsilon_n$. In the direct sum of the subgroups $\mathbb{Z}(p^n)$, this implies, successively, $z_1 = 0$ (modulo p), $z_2 - pz_1 = 0$ (modulo p^2), $z_3 - pz_2 = 0$ (modulo p^3), and so on. Inductively, this shows $z_n = 0$ (modulo p^n), n=1,2,..., and so $a = 0$. This completes the proof of the injectivity of η_p, and the remaining statements involved in the exactness of the sequence (E) are routine.

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page 689 In the diagram replace the leftmost letter "Z" by "$\mathbb{Z}$".

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page 694 Item (ii) of Corollary A1.43 read: $\cdots \mathbb{R}/\mathbb{Z} \cong \mathbb{Q}^{(\mathfrak{c})} \oplus \cdots$ (replace "=" by "$\cong$").

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page 705 Line 8 of proof of A1.61: Replace "$\dim_Q$" by "$\dim_{\mathbb{Q}}$".

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page 777 Lines 1, 2, 3 of Proposition A3.38 read as follows:

Proposition A3.38. *Assume that $F: \mathcal{A} \to \mathcal{B}$ and $U: \mathcal{B} \to \mathcal{A}$ are functors and $\eta: \mathrm{id}_{\mathcal{A}} \to UF$ and $\varepsilon: FU \to \mathrm{id}_{\mathcal{B}}$ are natural transformations. Then the following statements are equivalent:*

(1) *F is left adjoint to U and η and ε are the front adjunction and the back adjunction, respectively.*

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page 782 Line 3 from below: Replace "SAS" by "$\mathcal{A}$".

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page 782 Line 3 from below through page 783, line 3: Replace bold face type by roman type.

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page 787 Headline "Commutative...: Replace "its" by "their".

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page 794 Line 8 above Definition A3.66: Replace "group element" by "group object".

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page 817 Line 13: Replace "3.91" by "A3.91".

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page 853 line 853: Replace "measure" by "probability measure".

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page 858 Line 5 from below: Insert space between "of" and "X".

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page 864 Line 16 (counting headline) read: "compact" (not "coompact").

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page 878 Entry [219]: Delete one of two periods at the end of the line.

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page 886 Second column after X^α: Insert entry X_{fix}, **74**.

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page 900	Second column: Remove entry "generating degree"; in entry "generating rank" add "654, 658" after "**652**".
page 909	Second column: delete line "O(3), **255**".