Errata and Addenda in "The Structure of Compact Groups" 2nd Edition

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page 8	Lines 2, 3 of Exercise E1.6.(iii), read: $2\pi t + 2\pi \mathbb{Z} : \mathbb{T} \to \mathbb{R}/2\pi \mathbb{Z}.$		
	$R: \mathbb{R}/2\pi\mathbb{Z} \to \mathrm{SO}(2), R(t+2\pi\mathbb{Z}) = \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$		
page 15	Line 1 of Exercise E1.7, read: "For a discrete abelian group A and a compact abelian group G ".		
page 15	Line 17 from below, replace first " $q(t)$ by " $q(r)$ ".		
page 16	Line 14, replace " $\mu_n(\epsilon)$ " by μ_n .		
page 16	Line 2 from below: Delete " \langle " and " \rangle ".		
page 17	Line 13: Before the closing of "]" insert the following text: See [23], Chap. 7, §1, n ^o 3, Corollaire 2 de la Proposition 7.		
page 18	Lines 10, 11 read: "Then we obtain a projective system of compact groups by defining $f_{jj} = id_{G_j}$ and, for $j < k$ the morphisms".		
page 18	Line 18 from below: Replace " ϕ_p " by " ϕ_n ".		
page 21	Line 16 from below: Replace " $j \in J$ " by " $j \in F$ ".		
page 23	Line 4 from below: Replace " H " by " K ".		
page 28	Line 4: Replace " $\frac{\frac{1}{p^{\infty}}\mathbb{Z}}{\mathbb{Z}}$ " by " $\frac{\frac{1}{p^{\infty}}\mathbb{Z}}{p\mathbb{Z}}$ ".		
page 33	Line 9: Replace " $n \in N$ " by " $n \in \mathbb{N}$ ".		
page 39	Line 7: Replace " \int_g " by " \int_G ".		
page 92	Line 7, read: "partial information)." [Close parenthesis!]		

page 155	Line 13 from below: Replace "in the following proposition" by "now".
page 156	Line 6 read: (see A4.1ff.)
page 161	Line 6: Replace "exp \mathfrak{g} " by "exp: \mathfrak{g} "
page 173	Line 7 from below: in "close Lie subgroup" read "closed".
page 199	In Theorem 6.10 add new item (v) as follows:

(v) The Lie algebra \mathfrak{g} of G is an E-module under the adjoint action. If E contains an element h such that $\operatorname{Ad}(h)$ leaves no nonzero vector of g_{eff} fixed, then every element of $\operatorname{comm}(G, G_0)$ is a commutator.

page 200 Line 2 from below: Replace " $|G/G_0|$ " by "|E|".

page 202 Line 4, before Dinsert paragraph and the following:

(v) In the proof of (ii) in (13) we noted that $\operatorname{comm}(G, G_0) = \exp \mathfrak{g}_{\operatorname{eff}}$ where $\mathfrak{g}_{\operatorname{eff}} = \operatorname{span}\{\pi(e)X - X | e \in E, X \in \mathfrak{g}\}$. Now if $h \in E$ is such that $\pi(h)$ fixes no nonzero vector $X \in \mathfrak{g}_{\operatorname{eff}}$, then $\pi(h)|\mathfrak{g}_{\operatorname{eff}} - \operatorname{id}_{\mathfrak{g}_{\operatorname{eff}}} : \mathfrak{g}_{\operatorname{eff}} \to \mathfrak{g}_{\operatorname{eff}}$ is bijective and so for each $Y \in \mathfrak{g}_{\operatorname{eff}}$ there is an $X \in \mathfrak{g}_{\operatorname{eff}}$ such that $\exp Y = \exp(\operatorname{Ad}(h)X - X) =$ $\operatorname{comm}(h, \exp X)$.

page 203	Line 8: Remove space after "5.52(iii)".
page 203	Exercise E6.6 revise as follows:
	Replace the line preceding Exercise E6.6 by the following:

need not be closed as is discussed in the following exercise. First define the *com*mutator degree of a group P to be the smallest natural number n such that every element c of the commutator subgroup P' can be expressed in the form

 $c = \operatorname{comm}(p_1, q_1) \cdots (p_m, q_m), \text{ with } p_j, q_j \in P, 1 \le j \le m \le n.$

Exercise E6.6. (i) Fix a prime number $p \neq 2$. Let V_n denote an n-dimensional vector space over GF(p) and define a finite p-group $P_n = V_n \oplus \bigwedge^2 V_n$ with the multiplication $(x, v)(y, w) = (x + y, v + w + 2^{-1} \cdot (x \land y))$. Then the commutator degree of P_n is $\geq (n-1)/4$.

Set $G = \ldots$

Before "[Hint....]" insert the following:

(ii) Let n be a natural number ≥ 2 and let $E = \{1, -1\}^n$ for the multiplicative group $\{1, -1\}$ of integers. We identify E with its character group \hat{E} by setting, for

 $\chi = (\alpha_1, \ldots, \alpha_n), and e = (a_1, \ldots, a_n) in E,$

$$\langle \chi, e \rangle = \langle (\alpha_1, \dots, \alpha_n), (a_1, \dots, a_n) \rangle = \prod_{m=1}^n \alpha_m a_n \in \{1, -1\} \in \mathbb{S}^1.$$

Let $T \stackrel{\text{def}}{=} \mathbb{T}^{\hat{E} \setminus \{1\}}$ and let E act on T as follows:

$$t \cdot (x_{\chi})_{\chi \in \hat{E} \setminus \{1\}} = (\langle \chi, t \rangle \cdot x_{\chi})_{\chi \in \hat{E} \setminus \{1\}} \in T.$$

Let $\phi: E \to \operatorname{Aut} T$ be the associated morphism and define $P_n = T \rtimes_{\phi} E$. Then P_n is a metabelian Lie group and the commutator degree of P_n is $\geq 2^{n-1}$.

Set $G = \prod_{n=2}^{\infty} P_n$. Then G is a metabelian compact group in which $G' = \text{comm}(G, G_0)$ is not closed.

(iii) DAN SEGAL proposes a more systematic approach to (ii) above: Let E be any finite abelian group and $\mathbb{Z}[E]$ the discrete integral group ring of E. Then the additive group of $\mathbb{Z}[E]$ is free abelian on n generators, and so the ground ring extension $A \stackrel{\text{def}}{=} \mathbb{Z}[E] \otimes_{\mathbb{Z}} \mathbb{T}$ of the \mathbb{Z} -module T, as abelian group, is isomorphic to \mathbb{T}^n and thus carries the structure of an n-dimensional torus. The group $E \subseteq \mathbb{Z}(E)$ acts on the (commutative) ring $\mathbb{Z}[E]$ by multiplication and thus on A as a group of automorphisms. Thus $P = A \rtimes E$ is a compact metabelian group. If d is the cardinality of a minimal generating subset of E, then SEGAL shows that the commutator degree of P is d.

Let E_n , $n \in \mathbb{N}$, be any sequence of finite abelian groups such that the sequence of cardinalities d_n of a minimal generating set of E_n is unbounded. Set $P_n = (\mathbb{Z}(E_n) \otimes_{\mathbb{Z}} \mathbb{T}) \rtimes E_n$ and form $G = \prod_{n \in \mathbb{N}} P_n$. Then G is a metabelian compact group whose commutator subgroup is not closed.

page 203 Line 5 of Exercise E6.6: Replace "Hint." by "Hint. (i)".

page 203 Line 1 above Lemma 6.13: In front of "]" insert:

(ii) Let $\pi: E \to \operatorname{Gl}(\mathfrak{t})$ the representation of E associated with ϕ on the Lie algebra $\mathfrak{t} = \mathfrak{L}(T)$. Then the matrix of $\pi(e) - \operatorname{id}_{\mathfrak{t}}$ is diagonal with the diagonal entries $d_{\chi}, \chi \in \hat{E} \setminus \{1\}$, where

$$d_{\chi} = \begin{cases} 0 & \text{for } \langle \chi, e \rangle = 1, \\ -2 & \text{for } \langle \chi, e \rangle = -1 \end{cases}$$

Now let

$$N(e) = \operatorname{card}\{\chi \in E \setminus \{1\} : \langle \chi, e \rangle = 1\}.$$

Then dim $\operatorname{im}(\pi(e) - \operatorname{id}_{\mathfrak{t}}) = 2^{n} - 1 - N(e)$. The annihilator of $e \neq 1$ in the GF(2)-vector space E is a hyperplane and thus has dimension n-1 and therefore contains 2^{n-1} elements. Hence $N(e) = 2^{n-1} - 1$. Therefore

$$\dim \operatorname{im}(\pi(e) - \operatorname{id}_{\mathfrak{t}}) = 2^{n} - 1 - N(e) = 2^{n} - 1 - (2^{n-1} - 1) = 2^{n-1}.$$

As a consequence, the sum of M subspaces of the form $\operatorname{im}(\pi(e) - \operatorname{id}_{\mathfrak{t}})$ has a dimension $\leq M \cdot 2^{n-1}$. In order that this sum is equal to \mathfrak{t} , we have to have $2^n - 1 \leq 2^n - 1 < 2^n - 1$

 $M \cdot 2^{n-1}$, that is,

$$M \ge \frac{2^n - 1}{2^{n-1}} = 2^{n-1} - \frac{1}{2^{n-1}}.$$

and so $M \ge 2^{n-1}$ since M is a natural number and $1/2^{n-1} \le 1/2 < 1$.

Recalling the proof of part (ii) of 6.10 we conclude that this means that every element of G'_n is the product of no fewer than 2^{n-1} commutators.

The rest of the claim follows as in (i) above.

(iii) If A is a compact abelian group and E a finite group acting on A as a group of automorphisms, then in $G = A \rtimes E$ we have comm ((a, 1)(1, g))

- page 204f. From Theorem 6.15(iii) to the end the proof of 6.15: Replace " (G_0) " by " $(G')_0$ ". [In line 23 of page 205 the correct use is $\langle \exp \mathfrak{L}(G') \rangle = (G')_0$, while = $(G_0)'$ is incorrect.]
- page 205 Line 7: Replace the sentence starting with "Since" by the following: "By 5.54(iv) we have $\mathfrak{z}(\mathfrak{g}) = \mathfrak{z}(\mathfrak{L}(G_0) = \mathfrak{L}(Z(G_0))$. Since $Z(G) \cap G_0 \subseteq Z(G_0)$ we have $\mathfrak{L}(Z(G)) \subseteq \mathfrak{L}(Z(G_0)) = \mathfrak{z}(\mathfrak{g})$."
- page 217 Last line of the proof of 6.36: Replace "6.34" by "6.35".
- page 232 Line 13: Replace " \mathbb{R}^+ " by " R^+ ".
- page 234 Line 7: Replace " \mathbb{R}^+ " by " R^+ ".
- page 241 Line 17 from below: Replace " \mathbb{R}^+ " by " R^+ ".
- page 246 Line 1 of Theorem 6.52: Replace "compact" by "compact connected".
- page 246 Line 15 from below: Replace "acts" by "acts on".
- page 247 Line 1: Replace " $Ad(n_D)$ " by " $Ad(n_D)$ ".
- page 252 Line 3: Replace "= ad g" by "= ad \mathfrak{g} ".
- page 252 Lines 8, 9 read: " $\mathbb{G} = \operatorname{Aut} \mathfrak{g} \cong \operatorname{Aut}(\mathbb{R}^3, \times) = \operatorname{SO}(3)$." [Add "S" and omit the remainder.]
- page 252 Line 14: Replace " $e^{\operatorname{ad} X}$ " by " $e^{\operatorname{ad} X} = \operatorname{id}_{\mathfrak{q}}$ "
- page 255 Line 9 from below: Delete last parenthesis ")".
- page 262 Line 1 from below read: "Aut $\widetilde{G} \cong$ Aut $\mathfrak{g} =$ SO(3)."
- page 272 Line 1 in Theorem 6.74(II): Replace N(G,T) by N(T,G).

page 274	Line 4 below (*) read: $\psi_f \colon \tilde{P} \to P$.
page 275	Lines 4 and 1 above section headline: Replace " \rtimes_S " by " $\rtimes S$ ".
page 288	in the diagram replace " $(\frac{G}{T})$ " in a subscript by " $(\frac{G}{T}, K)$ ".
page 293	Line 3 above Postscript: Read " $\omega: \frac{G}{T} \times T \to G$ ".
page 298	Line 9: Replace "and (iii)" by "(iii) and (iv)".
page 322ff	Reference to the Literature:

The section on weakly complete vector spaces and their duality on pp. 322–331 (up to 7.31) is treated in a more detailed fashion in Appendix A2, pp. 628–649 of Hofmann, K. H., and S. A. Morris, The Lie Theory of Connected Pro-Lie Groups, EMS Publishing House, Zurich, 2007, xviii+676 pp.

page 327 Line 13 from below:Replace text from 'Thus in view of (i), (ii)...' to the end of the proof on page 328 by the following:

Thus in view of (i), (ii) above there is an isomorphism

$$\phi: \lim_{M \in \operatorname{Cofin}(E^*)} E^*/M \to \lim_{F \in \operatorname{Fin}(E)} F^*, \quad \phi\big((\omega_F + F^{\perp})_{F \in \operatorname{Fin}(E)}\big) = (\omega_F|F)_{F \in \operatorname{Fin}(E)}.$$

Now $\psi: E^* \to \lim_{F \in \operatorname{Fin}(E)} F^*$, $\psi(\omega) = (\omega|F)_{F \in \operatorname{Fin}(E)}$ is a morphism which is injective, because $E = \bigcup_{F \in \operatorname{Fin}(E)} F$, whence $\omega|F = 0$ for all F implies $\omega = 0$. Moreover, if $(\omega_F)_{F \in \operatorname{Fin}(E)} \in \lim_{F \in \operatorname{Fin}(E)} F^*$, then $F_1 \subseteq F_2$ in $\operatorname{Fin}(E)$ implies $\omega_{F_2}|F_1 = \omega_{F_1}$. Thus we may define unambiguously $\omega \in E^*$ by setting $\omega(x) = \omega_{\mathbb{R}\cdot x}(x)$ and obtain $\psi(\omega) = (\omega_F)_{F \in \operatorname{Fin}(E)}$. Then ψ is surjective and so is an isomorphism of vector spaces. We observe $\psi = \phi \circ \gamma_E$ and thus $\gamma_{E^*}: E^* \to \lim_{M \in \operatorname{Cofin}(E^*)} E^*/M$ is an isomorphism of vector spaces. Let $\operatorname{Fin}_1(E)$ be the set of one dimensional subspaces of E. Then $j: E^* \to \prod_{F \in \operatorname{Fin}(E)} \hat{F}$, $j(\omega) = (\omega|F)_{F \in \operatorname{Fin}(E)}$ is an embedding of topological vector spaces if E^* is given the weak *-topology. Consider the projection

$$\pi: \prod_{F \in \operatorname{Fin}(E)} \hat{F} \to \prod_{F \in \operatorname{Fin}_1(E)} \hat{F}, \quad \pi((\omega_F)_{\omega_F \in \operatorname{Fin}(E)}) = (\omega_F)_{F \in \operatorname{Fin}_1(E)}.$$

The diagram

is commutative. Since j is an embedding and $j \circ \psi^{-1} = \pi \circ$ incl is continuous, it follows that ψ^{-1} is continuous and so that ψ is an isomorphism of topological vector spaces. Hence γ_{E^*} is an isomorphism of topological vector spaces. This shows that

 E^* is a weakly complete topological vector space in the weak *-topology, and thus E' is weakly complete in the compact open topology by (i).

[The concepts 'compact open topology', and 'topology, compact open' go into the alphabetical index.]

page 328 Lines 1 and 2 after the proof of 7.28:

Replace line 1 and lines 2 through 'subset' by: Let E be a locally convex topological vector space over \mathbb{K} and E' its topological dual. If $\eta_E: E \to E'', \eta_E(x)(\omega) = \omega(x)$, denotes the evaluation morphism, then for each subset...

page 330	Line 20 from below:		
	Replace second half of line beginning with 'Thus $U^{\circ} \subseteq \cdots$ ' by the following:		
"Thus U° $\omega^{-1}B_{1/\epsilon}$. S	$ = (B_{\epsilon} \cdot \omega)^{\circ} \stackrel{\text{def}}{=} \{ x \in V : (B_{\epsilon} \cdot \omega)(x) \subseteq [0,1] \} = \{ x \in V : \omega(x) \le \frac{1}{\epsilon} \} = $ ince"		
page 349	The Section Heading should read: "Reducing Locally Compact Abelian Groups"		
page 374	Part (iii) of Corollary 8.9 has a shorter proof as follows: From 8.1(3) we have $G = \bigcup_{n \in \mathbb{N}} G[n]$. By the Baire Category Theo- rem, there is an n such that $G[n]$ has interior points and thus is an open closed subgroup of finite index. It readily follows that G has finite exponent that is, $G \subseteq G[N]$ for some $N \in \mathbb{N}$. This implies the assertion.		
page 404	Line 4ff.: Italicize the statement of Theorem 8.38.		
page 404	Line 8 from below read: "a vector subgroup E , where $(\operatorname{comp}(G))_{\ell}$ contains all torus subgroups and maps onto $G/\operatorname{comp}(G)$ "		
page 404	Lines 2, 1 from below read: " $(\operatorname{comp}(G))_{\ell}$ is locally connected, contains all torus subgroups, and maps onto $G/\operatorname{comp}(G)$."		
page 408	Item (ii) of Corollary 8.42 : Replace " \exp_G " by " \exp_{G_ℓ} "		
page 430	Line 5 from below, read:if and only if its rank does not exceed the cardinality of the continuum. Hence		
page 430	Line 3 from below: Insert period at the end of the line.		
page 431	Line 3 from below: Replace \mathbb{Z} by \mathbb{Z}_p .		

page 431 Line 1 from below: Replace Q by \mathbb{Q} .

page 450 Line 4ff. read: We showed in E6.6(ii) that it need not be closed in general.

page 450 Replace Exercise E9.2 by the following:

Exercise E9.2. Review the situation of compact groups and the circumstances in which the algebraic commutator subgroup is closed. (See Proposition 6.10, Theorem 6.11, Exercise E6.6, Theorem 6.18, Exercise E6.9, Theorem 6.55, Corollary 6.56, Theorem 9.2, Corollary 9.3.)

page 461	Line 1 read: $\widetilde{G}_{\mathfrak{s}} = S^{\aleph(\mathfrak{s},G)}_{[\mathfrak{s}]}$
page 473	Line 3 of $9.32(v)$ read:a connected abelian subgroup
page 476	Line 5 above Lemma 9.37: Delete line containing "Proof."
page 479	In Theorem 9.41: Delete the last line. [Covered by $(ii)'!$]
page 500	Line 14: Replace " SS^3 " by " \mathbb{S}^3 ".
page 501	Line 17: Replace "9.38" by "9.39".
page 502	Line 3: Read "seen in".
page 504	Line 10 from below: Twice replace " Z_p " by \mathbb{Z}_p ".
page 506	Line 8 of the Proof of 9.65: read " $(G_0)' \rtimes_{\iota} A$ ".
page 516	Line 6 from below: Replace "subgroup" by "group".
page 524	Line 14 : Replace "and all j " by "and all x ".
page 528	Line 4 : Replace \hat{Q}^X by $\hat{\mathbb{Q}}^X$.
page 556	Line 13: Replace "is a continuous function" by "yields a continuous function".
page 556	Line 24: Replace " ψ " by " Ψ ".
page 558	Line 16: Delete one of two consecutive periods.
page 558	Line 18 from below: Replace "1.51" by "10.31".

page 559	Line 2 of Corollary 10.37: Replace " G_0 " by " X_0 ".
page 560	Line 13 from below read: " $G_0 \times \mathbb{Z}(2)^{w(G/G_0)}$, where $w(\mathbb{Z}(2)^{w(G/G_0)}) = \dots$ "
page 561	Line 6 (including box) read: Exercise E10.8. Prove Theorem 10.41.
page 591	Line 2 from below read: $H^1(X, \mathbb{Z}) \cong [X, \mathbb{T}] = \{0\}$, then $FX \cong F_{ab}X \times F'X$. [2 corrections]
page 620	Line 12 from below: Replace " $s(X)$ " by " $s(G)$ ".
page 627	Line 13, replace "11.7." by 11.6(iii).".
page 639	Line 2 from below, replace two occurrences of " $j \in J$ " by " $x \in X$ ".
page 640	Line 2, replace " $n \in \mathbb{Z}$ " by " $n_x \in \mathbb{Z}$ ".
page 641	Line 5 (including box), replace " p (" by " p_x (".
page 641	Line 18, replace " F_x ." by " $F_x \cap G$.".
page 644	Line 10, read "complementary to P " (not B).
page 645	Line 4 from below: Replace " $\oplus e_{p+q} \oplus$ " by " $\oplus \mathbb{Z}.e_{p+q} \oplus$ ".
page 654	Line 10, replace " $n \mapsto n \cdot g_q$ " by " $n \mapsto n \cdot g_1$ ".
page 654	First Diagram, leftmost letter: Replace Z by \mathbb{Z} .
page 692	Proposition A2.8: In the Hint of the proof of Part (i), the continuity of the function $(x, y) \mapsto \psi_x(y)$ from $X \times Y$ to E is not rigorously established. A sufficient hypothesis for the completion of the proof is the local connectivity of X and Y .
page 703	Line 4, delete "Thus" and "and".
page 716	Line 8 from below, read: "such that $A = B \oplus \ker f$ and $B \cong B_1$, if and"
page 718	Line 4 from below, replace " X " by " Y ".
page 722	Line 19 from below read: " $\phi(v) = p_1(v) - p_2(v)$ "

- page 722 Line 15 from below: replace "equivariance" by "H-equivariant".
- page 724 Line 2 of A3.18, read: functions $D_j, R_j: C_j \to C_j$ and...
- page 727 Lines 18, 19: Replace "R" by " R_X "
- page 729 Diagram above line 6 from below: Replace \mathbb{C}_0 three times and C_0 once by \mathbb{H}_0 .
- page 734 Line 8 from below: Replace " $A_1 \rightarrow A_2$ " by " $FA_1 \rightarrow FA_2$ ".
- page 734 Line 5 from below: Replace " $\mathcal{A}(FA, B)$ " by " $\mathcal{B}(FA, B)$ ".
- page 740 Line 1: Replace " \mathcal{S} " by " \mathcal{C} ".
- page 740 Line 3: Replace "are C" by "are those of C".
- page 741 Line 16: Replace "map" by "morphism".
- page 741 Line 10 from below: Delete "exists".
- page 741 Line 8 from below: Replace " $q = \operatorname{pr}_A e$ " by " $q = \operatorname{pr}_B e$ ".
- page 746 Line 4: Replace "D(f, b)" by "D(f, B)".
- page 746 Line 8: Replace "an \mathcal{A} -morphism" by "a \mathcal{B} -morphism".
- page 746 Line 10: Replace " η_A, FA)" by " $\eta_A(FA)$ ".
- page 746 Line 11: Replace " \mathbb{A} " by A.
- page 746 Line 13: Replace "D(f, b)" by "D(f, B)".
- page 747 Lines 6, 7 read: " $\phi: FA \to B$ in \mathcal{B} such that $(U\phi)\eta_A = f$ in \mathcal{A} ."
- page 747 Line 7 from below: Replace " $g_i: k \to Ck_i$ " by " $g_i: k \to k_i$ "
- page 747 Line 5 from below: Replace " λ_{jf} " by " $\lambda_{j,f}$ ".
- page 748 Line 4: Replace " λ " by " $\tilde{\lambda}$ " twice.
- page 748 Line 3 of the proof of A3.59: Replace "A3.56(i)" by "A3.56(ii) and "(f, b)" by "(f, B)".

- page 748Line 14 from below: Replace " $UP \rightarrow M_i$ by $UP \rightarrow UM_j$ " (two
corrections!).page 748Line 10 from below: Replace " f_2, g_2 by " f_2g_2 ".page 754Diagrams above line 2 from below: Right diagram, the label of the
first vertical arrow should read id_E .page 800Line 2 of Definition A4.30 read: "cardinal N".
- page 800 Line 1 of Lemma A4.31: Delete "rm".
- page 840 contains an overfull hbox.