DEFINITIONS OF ONE-TO-ONE AND ONTO

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The first question on the algebra section of the 1978 Mathematics IA Exam at La Trobe University, asked for the definitions of *one-to-one* and *onto*, for a map $f : A \longrightarrow B$, using only symbols from the following list:

$$f,\ \ x,\ \ y,\ \ A,\ \ B,\ \ ),\ \ (,\ \ \in,\ \ \forall,\ \ \exists,\ \ =,\ \ \neq,\ \ \Rightarrow.$$

Below is a collection of students' answers. Exactly two of the "definitions" of one-to-one are correct, and exactly one of the "definitions" of onto is correct. Which are they?

	ONE-TO-ONE	*ONTO*
1.	$(\forall x \in A)(\forall y \in B)\ x = y \Rightarrow f(x) = f(y)$	$(\forall y \in A)(\exists x \in A)\ f(x) = y$
2.	$(\forall x \in A)(\forall y \in A)\ f(x) \neq f(y) \Rightarrow x \neq y$	$(\exists y \in B)(\forall x \in A)\ f(x) = y$
3.	$(\forall x \in A)(\exists y \in B)\ f(x) = y$	$(\forall x \in A)\ f(x) \in B$
4.	$\{(x \in a)(y \in B);\ x \Rightarrow y;\ y \Rightarrow x\}$	$f \Rightarrow B$
5.	$(\exists x \in A)(\exists y \in B)\ x = y$	$(\forall x \in A)(\forall y \in B)\ x \Rightarrow y$
6.	$(\forall x \in A)(\exists y \in A)\ f(x) = f(y)$	$(\forall y \in B) \wedge (\forall x \in A)\ f(x) = y$
7.	$(\forall x \in A)(\forall y \in B)\ f(x) \neq f(y)$ *and* $x \neq y$	$f(x) \neq y$
8.	$x \in A, y \in B\ \ i \neq j \Rightarrow y_i \neq y_j$	$x \in A, y \in B\ \forall y\ \exists x$ *such that* $x \rightarrow y$
9.	$(\forall x \in A)(\exists y \in B) \Rightarrow f(x_1) = y \neq f(x_2)$	$(\exists x \in A)(\forall y \in B) \Rightarrow f(x) = y$
10.	$(f(x) = y, x \in A, y \in B)(\forall x \Rightarrow y)(\forall y \Rightarrow \exists x)$	$(f(x)=y,\ x \in A, y \in B)(\forall x \Rightarrow \exists y)(\forall y \not\Rightarrow \exists x)$
11.	$(\forall x \in A)(\forall y \in B)\ x = y \Rightarrow y = x$	$(\forall x \in A)(\forall y \in B)\ x \neq y \Rightarrow y \neq x$
12.	$(\exists x \in A)(\exists y \in B),\ f(x,y)$	$(\forall x \in A)(\exists y \in B),\ f(x,y)$
13.	$(\forall x \in A)(\forall y \in B)\ x = y \Rightarrow f(x) = f(y)$	$(\forall x \in A)(\exists y \in B)\ f(y) = x$
14.	$(\forall x \in A)(\forall x' \in A)\ f(x) \neq f(x')$	$(\forall y \in B)(\exists x \in A)\ f(x) = y$
15.	$\forall x \in A\ f(x) \Rightarrow y \in B$	$\forall x \in A\ f(x) \Rightarrow y \in B : A = B$
16.	$(\forall x \in A)(\forall y \in B)\ f(x) = f(y) \Rightarrow x = y$	$f(x) = y$
17.	$(\forall y)(\forall x)\ f(x) = f(y)\ \ \ x = y$	$(\forall y)(\exists x)\ \ f(x) = y$
18.	$f(x) \neq f(y),\ x \neq y$	$(\exists y \in B)(\forall x \in A)\ \ f(y) = x$
19.	$(\forall x, y \in A) = (\forall x, y \in B)$	$(\forall x, y \in A)(\exists x, y \in B)$
20.	$(\forall y \in B)(\forall x \in A)\ x \neq y \Rightarrow f(x) \neq f(y)$	$(\forall y \in A)(\exists x \in B)\ f(x) = y$
21.	$f : (\exists x \in A)(\exists y \in B)\ \ x \Rightarrow y$	$f : (\exists x \in A)(\forall y \in B)\ \ x \Rightarrow y$
22.	$x \Rightarrow Af(x)\ \ \ x \Rightarrow Bf(x)\ \ \ A \neq B$	$(\exists x)(\forall y)\ \ x \Rightarrow y$
23.	$\exists(x)(x \in A)$ *and* $\exists(y)(y \in B)\ x \Rightarrow y$	$\forall(x)(x \in A)$ *and* $\forall(y)(y \in B)\ x \Rightarrow y$
24.	$(\forall x \in A)(\forall y \in B)\ f(A) \neq f(B) \Rightarrow A \neq B$	$(\forall y \in A)(\exists x \in B)\ \ f(x) = y$
25.	$(\forall x \in A)(\exists y \in B)\ \ y = f(x)$	$y = f(x)\ \ (\forall y \in B)(\exists x \in A)$

26.	$(\forall x \in A)(\forall y \in A)\ x \neq y \Rightarrow f(x) \neq f(y)$	$(\forall x \in B)(\exists y \in B)\ f(x) = y$
27.	$(\forall y \in A)(\forall x \in A)\ x \neq y \ \&\ f(x) \neq f(y)$	$f : A \longrightarrow B,\ f = B$
28.	$\exists x \Rightarrow \exists y$	$\exists x \Rightarrow \forall y$
29.	$\forall x \in A,\ \forall y \in B,\ f(x) = f(y)\ \forall x \neq \exists y$	$x \in A \quad y \in B \quad \forall y \Rightarrow \exists x$
30.	$(\forall x \in A)\ (\forall y \in B)\quad x \Rightarrow y$	$(\forall x \in A)(\exists y \in B)\quad x \Rightarrow y$
31.	$x \neq y \ \&\ A \neq B$	$(\forall x \in A)(\exists y \in B)$
32.	$(\forall x \in A)(\forall y \in A)\ f(x) = f(y) \Rightarrow x = y$	$\forall x,y \in A\quad f(x),\ f(y) \in B$
33.	$(\forall x \in A)(\forall y \in B)\ x = y \neq 2x + y$	$(\forall y \in B)(\exists x \in A)\ x = y \neq x + y$
34.	$\{f : x \neq y,\ f(x) \neq f(y)\}$	$\{f : (\forall x \in A)(\exists y \in B) : A \longrightarrow B\}$
35.	$f : A\ \forall B$	$f : A \Rightarrow B$
36.	$(\forall y)(\forall x)\ y = f(x)\ y \neq x$	$(\forall y)(\exists x)\quad y = f(x)$
37.	$(\forall y \in B)(\exists x \in A)\ y = x$	$(\exists x \in A)(y \in B)$
38.	$(\forall A \in f)(\exists B \in f)\ x = y$	$(\forall A \in f)(\forall B \in f)\quad x \Rightarrow y$
39.	$\exists x \in A,\ \exists y \in B \Rightarrow f(x) = y$	$\forall x \in A,\ \forall y \in B \Rightarrow f(x) = y$
40.	$(\forall x \in A)(\exists y \in B)\ x \Rightarrow y$	$(\exists x \in A)(\forall y \in B)\ y \Rightarrow x$
41.	$(\forall x \in A)(\forall y \in A)\ x = y \Rightarrow f(x) = f(y)$	$(\forall y \in A)(\exists x \in A)\quad f(x) = y$
42.	$(\forall x \in A)(\exists y \in B)\ f : x = y$	$(\exists x \in A)(\forall y \in B)\ f : x = y$
43.	$(\exists x \in A)(\exists y \in B)\quad f(x) = y$	$(\forall x \in A)(\forall y \in B)\ f(x) = y$
44.	$[(\forall x \in A)(\forall y \in B)f(x) = f(y)] \Rightarrow [x = y]$	$(\forall x \in A)(\exists y \in B)\ x = y$
45.	$(\forall x,y \in A)\ f(x) = y$	$(\exists x \in A)(\forall x \in A)\ f(x) = f(y) \Rightarrow x = y$
46.	$\exists x \in A\ \exists y \in B\ x \Rightarrow f(x) = y$	$\exists x \in A\ \forall y \in B\ x \Rightarrow f(x) = y$

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