Free products of topological groups

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In this note the notion of a free topological product $\prod^* G_\alpha$

of a set $\{G_\alpha\}$ of topological groups is introduced. It is

shown that it always exists, is unique and is algebraically

isomorphic to the usual free product of the underlying groups.

Further if each G_α is Hausdorff, then $\prod^* G_\alpha$ is Hausdorff

and each G_α is a closed subgroup. Also $\prod^* G_\alpha$ is a free

topological group (respectively, maximally almost periodic) if

each G_α is. This notion is then combined with the theory of

varieties of topological groups developed by the author. For $\underline{V}$

a variety of topological groups, the $\underline{V}$-product of groups in $\underline{V}$

is defined. It is shown that the $\underline{V}$-product, $\underline{V} \prod G_\alpha$ of any

set $\{G_\alpha\}$ of groups in $\underline{V}$ exists, is unique and is

algebraically isomorphic to the usual varietal product. It is

noted that the $\underline{V}$-product of Hausdorff groups is not necessarily

Hausdorff, but is if $\underline{V}$ is abelian. Each G_α is a quotient

group of $\underline{V} \prod G_\alpha$. It is proved that the $\underline{V}$-product of free

topological groups of $\underline{V}$ and projective topological groups of

$\underline{V}$ are of the same type. Finally it is shown that $\underline{V} \prod G_\alpha$ is

connected if and only if each G_α is connected.

1. Introduction

In this note we introduce the notion of a free topological product of

topological groups. The concept is the natural analogue of that of a free
product of groups [5] and was inspired by the work of Golema [2] and
Hulanicki [7] on free products in the category of compact Hausdorff groups.

In §2 we prove basic existence and uniqueness theorems for the free
topological product and the free abelian topological product and ape some
theorems on free products. We note that Hulanicki's result (Proposition 1
of [7]) can be interpreted as saying the free topological product of
compact Hausdorff groups is maximally almost periodic [15]. We extend
this result to show that the free topological product of maximally almost
periodic groups is of the same type. Hall [4] noted that the direct
product of projective topological groups is not necessarily a projective
topological group. In the light of this it is satisfying to see that the
free abelian topological product of projective topological groups is
projective.

In §3 we combine the notion of a free topological product with that of
a variety of topological groups; (see [10], [11] and [12]). We define a
$\underline{V}$-product of topological groups as a natural analogue of a varietal product
[14]. Similar results are then proved for $\underline{V}$-products.

The notation and terminology, in general, conform to that of [8] and
[14]. Further if G is a topological group then $\overline{G}$ denotes the
underlying (abstract) group. If S is any set then $|S|$ denotes the
cardinal of the set. If A and B are topological groups then A is a
subgroup of B , means that A is algebraically a subgroup of B and that
the topology induced on A by B is the given topology. The cartesian
product of topological groups $\{G_i : i \in I\}$ is always given the Tychonoff
product topology and denoted by $\prod_{i \in I} G_i$.

2. Free topological products

DEFINITION 2.1. Let $\{G_i : i \in I\}$ be a set of topological groups.
Then the topological group F is said to be a free topological product of
$\{G_i : i \in I\}$, denoted by $\prod_{i \in I}^* G_i$, if it has the properties:

(a) for each i in I , G_i is a subgroup of F ;

(b) F is generated algebraically by $\bigcup\limits_{i \in I} G_i$;

(c) if for each i in I , ϕ_i is a continuous homomorphism of G_i
into a topological group H , then there exists a continuous
homomorphism Φ of F into H such that $\Phi = \phi_i$ on G_i , for
each i .

THEOREM 2.2. *Let* $\{G_i : i \in I\}$ *be any set of Hausdorff groups.*
Then $\prod\limits_{i \in I}^* G_i$ *exists and is Hausdorff.*

Proof. Let $\overline{F} = \prod\limits_{i \in I}^* \overline{G}_i$ be the free (algebraic) product of the
groups $\{\overline{G}_i : i \in I\}$. (See [5].) Our first task is to show that $\overline{F}$ can
be given a Hausdorff group topology which induces the given topology on
each G_i .

We define a topology τ_1 on $\overline{F}$ by exhibiting a system of
left-invariant pseudo-metrics d ([8], p. 210) each of which is given by
a family $\{d_i : i \in I\}$, where d_i is a left-invariant pseudo-metric on
G_i for each i in I .

For g any element of $\overline{F}$, $g = g_{\alpha_1} \cdots g_{\alpha_n}$, where $g_{\alpha_i} \in G_{\alpha_i}$ and

$\alpha_i \neq \alpha_{i+1}$. Define $d(g, e) = \sum\limits_{i=1}^{n} d_{\alpha_i}(g_{\alpha_i}, e)$, where e represents the

identity element of $\overline{F}$ and each G_i . Then for h and k any elements

in $\overline{F}$, define $d(h, k) = d(k^{-1}h, e)$. It is easily seen that d is a
left-invariant pseudo-metric on $\overline{F}$ and if g is an element of $\overline{F}$ such
that $d(g, e) = 0$ for all d , then $g = e$. Thus the topology τ_1 on $\overline{F}$
induced by the family of all d is a Hausdorff group topology. Clearly
τ_1 induces the given topology on each G_i .

Now define F to be the group $\overline{F}$ with the finest group topology τ
which induces the given topology on each G_i . From the previous paragraph
we see that F exists and is Hausdorff. We claim that F is the required

group. (Clearly it has properties (a) and (b) of Definition 2.1.)

Let H be any topological group and ϕ_i , for each i in I , be a continuous homomorphism of G_i into H . Since F is algebraically a free product of the G_i , there is a homomorphism Φ of F into H such that $\Phi = \phi_i$ on G_i , $i \in I$. Define a topology τ_2 on $\overline{F}$ in the following manner: the set O is open in τ_2 if and only if $O = \Phi^{-1}(A)$, where A is an open set in H . Clearly τ_2 is a group topology. Let the topology τ_3 be the sum of τ_2 and τ ; ([3], §1). Then $\overline{F}$ with the topology τ_3 is a topological group and τ_3 induces the given topology on each G_i . Since τ_3 is finer than τ we must have $\tau_3 = \tau$. Hence Φ is continuous, and the proof is complete.

The next theorem shows that the free topological product is unique (if it exists). The proof is omitted as it is similar to that of Theorem 8.9 of [6].

THEOREM 2.3. *Let* $\{G_i : i \in I\}$ *be a set of topological groups. If* F_1 *and* F_2 *both satisfy conditions* (a), (b) *and* (c) *of Definition 2.1, then there is a topological isomorphism* f *of* F_1 *onto* F_2 *such that the mapping* f *restricted to* G_i *($\subseteq F_1$) is a topological isomorphism of* G_i *onto* G_i *($\subseteq F_2$) for each* $i \in I$.

COROLLARY 2.4. *If* $\{G_i : i \in I\}$ *is a set of Hausdorff groups, then* $\prod_{i \in I}^* G_i$ *is algebraically isomorphic to the free (algebraic) product of* $\{\overline{G_i} : i \in I\}$.

THEOREM 2.5. *Let* $\{G_i : i \in I\}$ *be a set of Hausdorff groups and* J *be a subset of* I . *Then the subgroup* G *of* $\prod_{i \in I}^* G_i$ *generated algebraically by* $\bigcup_{i \in J} G_i$ *is topologically isomorphic to* $\prod_{i \in J}^* G_i$. *Further,* G *is a closed subgroup of* $\prod_{i \in I}^* G_i$.

Proof. In the notation of Theorem 2.2, it is clear that G is a

closed subset in τ_1 . Since τ is finer than τ_1 , G is a closed

subgroup of $\prod_{i \in I}^{*} G_i$.

Let H be any topological group and ϕ_i a continuous homomorphism

of G_i into H for each i in J . For each i in I which is not in

J , define a continuous homomorphism ϕ_i of G_i into H by

$\phi_i(G_i) = e$, the identity of H . Then there is a continuous homomorphism

Φ of $\prod_{i \in I}^{*} G_i$ into H such that $\Phi = \phi_i$ on G_i . Restrict Φ to G .

Clearly then, condition (c) of Definition 2.1 is satisfied. Hence G is

the free topological product of $\{G_i : i \in J\}$.

COROLLARY 2.6. *If* $\{G_i : i \in I\}$ *is a set of Hausdorff groups then*

each G_i *is a closed subgroup of* $\prod_{i \in I}^{*} G_i$.

The following lemma can be proved in precisely the same way as Lemma
2.11 of [10].

LEMMA 2.7. *Let* X *be any subset of a topological group* H . *If* Φ
is a homomorphism of H *into a topological group* G *such that* $\Phi|X$ *is*
an open mapping, then Φ *is an open mapping.*

THEOREM 2.8. *Let* $\prod_{i \in I}^{*} G_i$ *be the free topological product of a set*

$\{G_i : i \in I\}$ *of topological groups. Then each* G_i *is a quotient group*

of $\prod_{i \in I}^{*} G_i$.

Proof. Let j be a fixed element of I . For each i in I ,
$i \neq j$, define a mapping ϕ_i of G_i into G_j by $\phi_i(G_i) = e$. Define

ϕ_j to be the identity mapping of G_j into itself. Then each ϕ_i ,

$i \in I$, is a continuous homomorphism. Therefore, there is a continuous

homomorphism Φ of $\prod_{i \in I}^{*} G_i$ into G_j such that $\Phi|G_i = \phi_i$, for each i

in I .

Now $\Phi|G_j$ is an open mapping. Thus by Lemma 2.7, Φ is an open mapping, and the proof is complete.

THEOREM 2.9. *Let* $F(X_i)$ *be a free topological group on a topological space* X_i *, for* i *in an index set* I *. ([9], or [10], Definition 4.1.) Then* $\displaystyle\prod_{i \in I}^* F(X_i)$ *exists and is* $F(Y)$ *, the free topological group on the space* Y *, where* Y *is the free union [1] of the spaces* $\{X_i : i \in I\}$ *.*

Proof. By Theorem 6.5 of [12], $F(Y)$ exists. It is easily verified that the subgroup of $F(Y)$ generated algebraically by X_i is $F(X_i)$, for each i in I .

Let ϕ_i be a continuous homomorphism of $F(X_i)$ into a topological group H , for each i in I . Then $\phi_i|X_i$ is a continuous mapping of X_i into H for each i . Define a map ϕ of Y into H by $\phi|X_i = \phi_i|X_i$, for each i . Then ϕ is continuous. Thus there is a continuous homomorphism Φ of $F(Y)$ into H such that $\Phi|Y = \phi$. Hence $\Phi|X_i = \phi_i$ and therefore $\Phi|F(X_i) = \phi_i$, for each i . Thus $F(Y)$ is

$$\prod_{i \in I}^* F(X_i) .$$

DEFINITION 2.10. A topological group G is said to be maximally almost periodic (MAP) , if it admits a continuous monomorphism into a compact Hausdorff group; (see [15]).

For example every compact Hausdorff group, and every locally compact Hausdorff abelian group is a MAP group.

THEOREM 2.11. *If* $\{G_i : i \in I\}$ *is a set of* MAP *groups, then* $\displaystyle\prod_{i \in I}^* G_i$ *is a* MAP *group.*

Proof. For each i in I , there is a compact Hausdorff group H_i and a continuous monomorphism ϕ_i of G_i into H_i . Proposition 1 of

[7] implies that $\prod_{i \in I}^{*} H_i$ is a MAP group.

Furthermore there is a continuous homomorphism Φ of $\prod_{i \in I}^{*} G_i$ into $\prod_{i \in I}^{*} H_i$ such that $\Phi|G_i = \phi_i$, for each i. Clearly Φ is a monomorphism. Hence $\prod_{i \in I}^{*} G_i$ is a **MAP** group.

REMARK 2.12. If we restrict our attention to abelian groups we can obtain similar results. In fact if we take all groups previously discussed in this section to be abelian (and hence define a free abelian topological product) all the results remain true.

The following theorem is obvious.

THEOREM 2.13. *If* $\{G_i : i \in I\}$ *is any set of abelian topological groups then their free abelian topological product exists and has the direct product* $\prod_{i \in I}^{D} G_i$ *of these groups as a continuous monomorphic image.*

REMARK 2.14. Remark 3.32 of [13] states that the direct product of free abelian topological groups is not necessarily a free abelian topological group. Thus the analogue for abelian groups of Theorem 2.9, which states that the free abelian topological product of free abelian topological groups is a free abelian topological group, is of interest.

Also Theorem 2.15 is notable, since Example 2 of [4] shows that the direct product of projective topological groups is not necessarily projective.

THEOREM 2.15. *Let* $\{G_i : i \in I\}$ *be a set of projective topological groups (in the sense of [4]). Then the free abelian topological product F of* $\{G_i : i \in I\}$ *is a projective topological group.*

Proof. Let A and B be abelian topological groups and f be a continuous homomorphism of A onto B such that there exists a continuous mapping g of B into A such that fg acts identically on B.

Let θ be a continuous homomorphism of F into B and $\theta_i = \theta|G_i$ for each i in I . Then θ_i is a continuous homomorphism of G_i into B . Since each G_i is projective, there is a continuous homomorphism ϕ_i of G_i into A such that $f\phi_i = \theta_i$. Therefore there is a continuous homomorphism Φ of F into A such that $\Phi|G_i = \phi_i$. Clearly $f\Phi = \theta$. Hence F is projective.

3.   Topological varietal products

DEFINITION 3.1.   Let $\underline{V}$ be a variety of topological groups [10] and $\{G_i : i \in I\}$ be a set of topological groups in $\underline{V}$. Then the topological group F in $\underline{V}$ is said to be a $\underline{V}$-product of $\{G_i : i \in I\}$, denoted by $\underline{V}\prod_{i \in I} G_i$, if it has the properties:

(a)   each G_i , $i \in I$, is a subgroup of F ;

(b) F is generated algebraically by $\underset{i \in I}{\cup} G_i$;

(c)   if for each i in I , ϕ_i is a continuous homomorphism of G_i into a topological group H in $\underline{V}$, then there exists a continuous homomorphism Φ of F into H such that $\Phi = \phi_i$ on G_i , for each i .

Clearly, if $\underline{V}$ is the variety of all (respectively, all abelian) topological groups, then $\underline{V}\prod_{i \in I} G_i$ is the free (free abelian) topological product of $\{G_i : i \in I\}$.

THEOREM 3.2.   *Let* $\{G_i : i \in I\}$ *be any set of topological groups in a variety of topological groups* $\underline{V}$. *Then* $\underline{V}\prod_{i \in I} G_i$ *exists.*

Proof.   Let m be any infinite cardinal such that $\left|\underset{i \in I}{\cup} G_i\right| \leq m$. Let $\{H_\alpha : \alpha \in M\}$ be the set of all (non-topologically isomorphic) topological groups in $\underline{V}$ such that $|H_\alpha| \leq m$. We denote by β a system

of continuous homomorphisms $f_i^{\alpha\beta}$, $i \in I$ of the G_i into a fixed H_α .
The set of all such systems may be denoted by N_α . Let $G = \prod\limits_{\alpha,\beta} H_{\alpha\beta}$,

with $H_{\alpha\beta} = H_\alpha$ for $\beta \in N_\alpha$.

For each i in I , define the map σ_i of G_i into G by

$\sigma_i(x) = \left\{ f_i^{\alpha\beta}(x) \right\}$. Clearly by the embedding lemma in [8], each σ_i is a
topological isomorphism of G_i onto $\sigma_i(G_i)$. We now identify G_i with
$\sigma(G_i)$.

Let F be the subgroup of G generated algebraically by $\bigcup\limits_{i \in I} G_i$.
Then F is the required group. We see that F has properties (a) and
(b) of Definition 3.1.

Let ϕ_i be a continuous homomorphism of G_i into a topological
group H in $\underline{V}$ for each i in I . Without loss of generality we
can assume $|H| \leq m$. Thus for some α and β we can identify H with
$G_{\alpha\beta}$ and ϕ_i with $f_i^{\alpha\beta}$ for each i . Hence, if we define Φ to be the
projection of F onto the axis corresponding to the indices α, β we get
$\Phi | G_i = \phi_i$, for each i in I . The proof is complete.

The following uniqueness theorem can be proved in a similar manner to
Theorem 8.9 of [6].

THEOREM 3.3. *Let* $\{G_i : i \in I\}$ *be a set of topological groups in a*
variety of topological groups $\underline{V}$. *If* F_1 *and* F_2 *are in* $\underline{V}$ *and have*
the properties stated in Definition 3.1, then there is a topological
isomorphism f *of* F_1 *onto* F_2 *such that the mapping* f *restricted to*
$G_i \left(\subseteq F_1\right)$ *is a topological isomorphism of* G_i *onto* $G_i \left(\subseteq F_2\right)$ *for each*
i .

THEOREM 3.4. *Let* $\underline{V}$ *be a variety of topological groups and* $\overline{\underline{V}}$ *be*
the underlying (algebraic) variety of groups. *(See Remark 2.3 of [10].)*
If $\{G_i : i \in I\}$ *is a set of topological groups in* $\underline{V}$, *then* $\underline{V} \overline{\prod\limits_{i \in I} G_i}$

is the $\underline{\underline{V}}$*-product of* $\{\overline{G}_i : i \in I\}$. (See [14], 18.31.)

Proof. For each i in I , let ϕ_i be a homomorphism (not necessarily continuous) of G_i into any H in $\underline{\underline{V}}$. It is enough to prove that there is a homomorphism Φ of $\underline{\underline{V}} \prod_{i \in I} G_i$ into H such that $\Phi|G_i = \phi_i$ for each i .

By Lemma 2.7 of [10], there is an A in $\underline{\underline{V}}$ which has the indiscrete topology and is algebraically isomorphic to H . Let θ be an isomorphism of H onto A . Then for each i , the mapping $\theta\phi_i$ is a continuous homomorphism of G_i into A . Therefore, there is a continuous homomorphism δ of $\underline{\underline{V}} \prod_{i \in I} G_i$ into A such that $\delta|G_i = \theta\phi_i$, for each i . Define the mapping Φ of $\underline{\underline{V}} \prod_{i \in I} G_i$ into H by $\Phi = \theta^{-1}\delta$. Clearly Φ is a homomorphism and $\Phi|G_i = \phi_i$ for each i .

REMARK 3.5. Theorem 2.2 leads us to ask: If $\{G_i : i \in I\}$ is a set of Hausdorff groups in a variety of topological groups $\underline{\underline{V}}$, is $\underline{\underline{V}} \prod_{i \in I} G_i$ necessarily Hausdorff? The following example answers this question in the negative, whilst Theorem 3.7 shows that if $\underline{\underline{V}}$ is an abelian variety then the answer is positive.

EXAMPLE 3.6. (See [12], Example 3.2.) Let $\underline{\underline{V}}$ be the class of all topological groups G having the property that the intersection of all neighbourhoods of the identity in G contains the commutator subgroup of G . It can be readily verified that $\underline{\underline{V}}$ is a variety of topological groups such that the underlying (algebraic) variety of groups is the variety of all groups.

Clearly the infinite cyclic group F with the discrete topology is in $\underline{\underline{V}}$. However, by Theorem 3.4, the $\underline{\underline{V}}$-product of F with itself is a non-abelian group and hence is not Hausdorff.

THEOREM 3.7. *Let* $\underline{\underline{V}}$ *be any variety of abelian topological groups and* $\{G_i : i \in I\}$ *be any set of Hausdorff groups in* $\underline{\underline{V}}$ *. Then* $\underline{\underline{V}} \prod_{i \in I} G_i$

is a Hausdorff group and each G_i is a closed subgroup.

Proof. This follows immediately from the fact that the direct product of $\{G_i : i \in I\}$ is in $\underline{V}$, is a Hausdorff group which is algebraically isomorphic to $\underline{V} \prod_{i \in I} G_i$, and has each G_i as a closed subgroup.

The next theorem is obvious.

THEOREM 3.8. *Let $\underline{V}$ be any variety of topological groups and $\{G_i : i \in I\}$ a set of topological groups in $\underline{V}$. Consider the sequence*

$$\prod_{i \in I}^{*} G_i \xrightarrow{f} \underline{V} \prod_{i \in I} G_i \xrightarrow{g} \prod_{i \in I}^{D} G_i$$

where $\prod_{i \in I}^{D} G_i$ is the direct product of the G_i and the mappings f and g are the natural homomorphisms. Then f and g are continuous. Further, if $\underline{V}$ is a full variety [11], f is an open mapping.

Theorems 3.9, 3.10, 3.11 and 3.12 can be proved in a similar manner to Theorems 2.8, 2.9, 2.15 and 2.5 respectively.

THEOREM 3.9. *Let $\underline{V}$ be a variety of topological groups and $\{G_i : i \in I\}$ be in $\underline{V}$. Then each G_i is a quotient group of $\underline{V} \prod_{i \in I} G_i$.*

THEOREM 3.10. *Let $\underline{V}$ be a variety of topological groups and $F(X_i, \underline{V})$, $i \in I$, be a set of free topological groups of $\underline{V}$ [10]. Let $Y = \underset{i \in I}{\cup} X_i$ in $\underline{V} \prod_{i \in I} F(X_i, \underline{V})$. Then $\underline{V} \prod_{i \in I} F(X_i, \underline{V})$ is $F(Y, \underline{V})$.*

THEOREM 3.11. *Let $\underline{V}$ be a variety of topological groups and $\{G_i : i \in I\}$ be a set of topological groups projective in $\underline{V}$ [11]. Then $\underline{V} \prod_{i \in I} G_i$ is projective in $\underline{V}$.*

Note that Theorem 3.6 of [11] is a special case of Theorem 3.11.

THEOREM 3.12. *Let $\{G_i : i \in I\}$ be a set of topological groups in*

a variety of topological groups $\underline{V}$. *Let* J *be a subset of* I . *Then the subgroup of* $\underline{V} \prod_{i \in I} G_i$ *generated algebraically by* $\underset{i \in J}{U} G_i$ *is topologically isomorphic to* $\underline{V} \prod_{i \in J} G_i$.

THEOREM 3.13. *Let* $\underline{V}$ *be a variety of topological groups and* $\{G_i : i \in I\}$ *be in* $\underline{V}$. *Then* $\underline{V} \prod_{i \in I} G_i$ *is a connected group if and only if each* G_i *is connected.*

Proof. If $\underline{V} \prod_{i \in I} G_i$ is connected, then it follows immediately from Theorem 3.9 that each G_i is connected.

Conversely if each G_i is connected, then $\underset{i \in I}{U} G_i$ is a connected subset of $\underline{V} \prod_{i \in I} G_i$ containing the identity. Further $\underset{i \in I}{U} G_i$ generates $\underline{V} \prod_{i \in I} G_i$ algebraically. Hence $\underline{V} \prod_{i \in I} G_i$ is connected.

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Free products of topological groups: Corrigendum

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Professor Edward T. Ordman has pointed out to the author that the proof of Theorem 2.2 in [2] is incorrect. The theorem is, in fact, correct and was proved by Graev [1]. The incorrect proof presented in my paper has been modified by Ordman [3] to provide a much simpler proof than Graev's in a special case.

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